



ME 327: Design and Control of Haptic Systems

Spring 2020

Interactive Session 10: Kinesthetic haptic devices: Stability

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Questions from
prerecorded video?

effect of time delay

review stability in the context of the s-plane

common second-order system: $m\ddot{x} + b\dot{x} + kx = f$

take the Laplace transform of both sides:

$$\mathcal{L}[m\ddot{x} + b\dot{x} + kx] = \mathcal{L}[f]$$

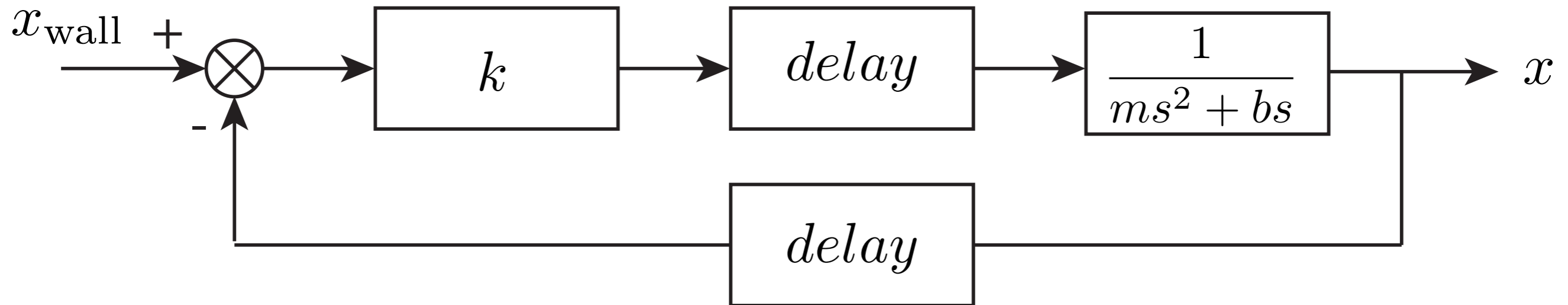
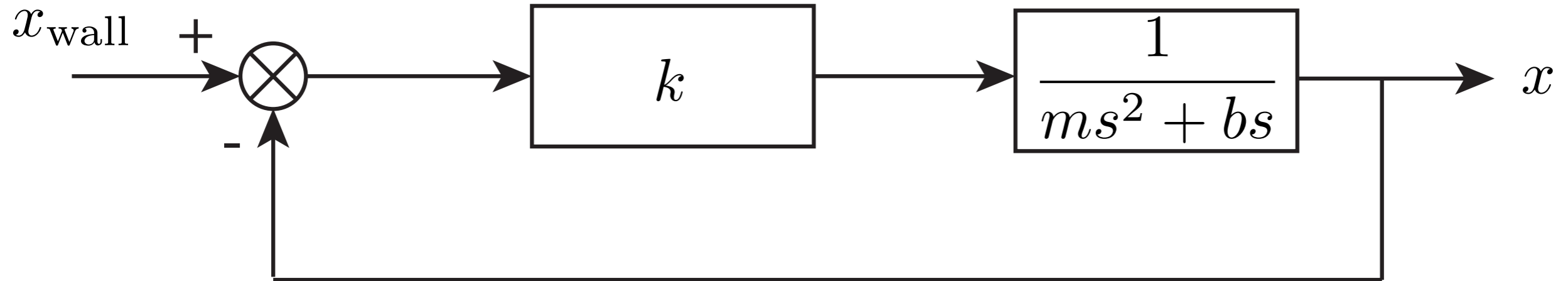
$$ms^2 X(s) + bsX(s) + kX(s) = F(s)$$

$$(ms^2 + bs + k)X(s) = F(s)$$

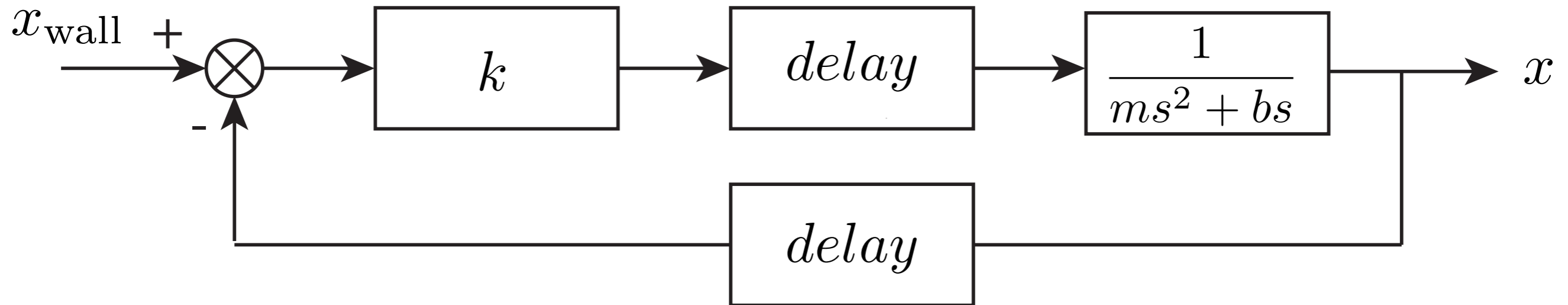
transfer function/characteristic equation:

$$\frac{F(s)}{X(s)} = ms^2 + bs + k$$

Time Delay



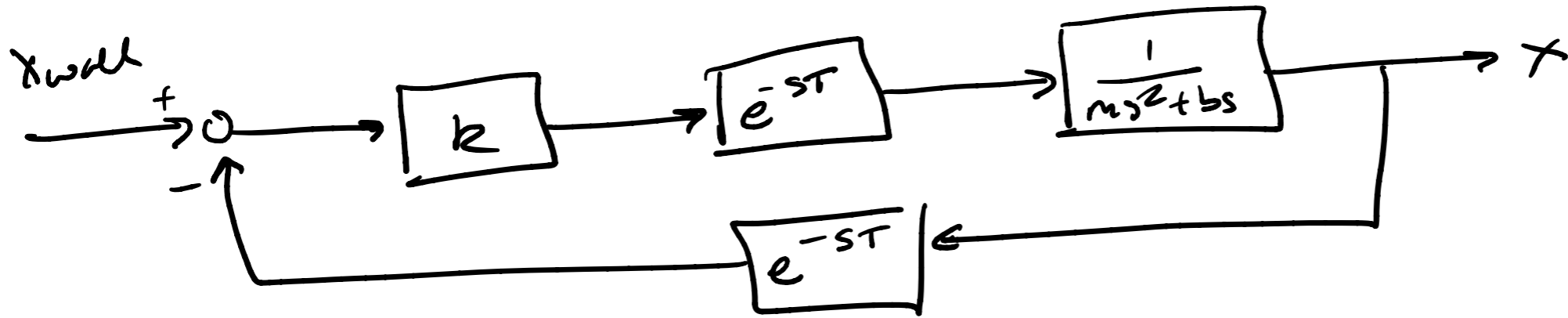
Padé approximation



$$\text{delay} = e^{-sT} \quad e^{-sT} \approx \frac{1 - \left(\frac{sT}{2}\right)}{1 + \left(\frac{sT}{2}\right)}$$

This adds a left half plane pole and a right half plane zero!

In breakout groups: Discuss/calculate how time delay affects the maximum stable stiffness of a virtual wall



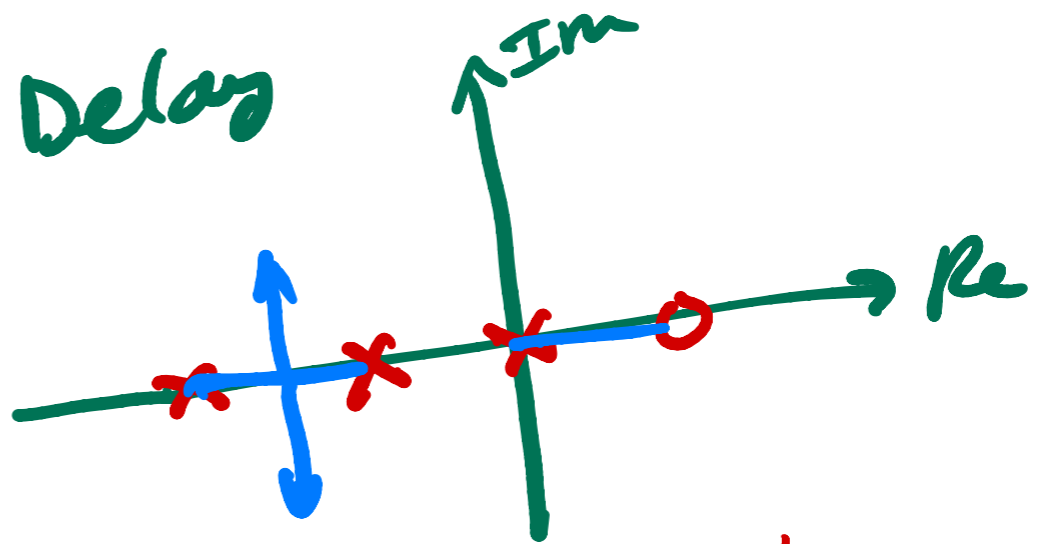
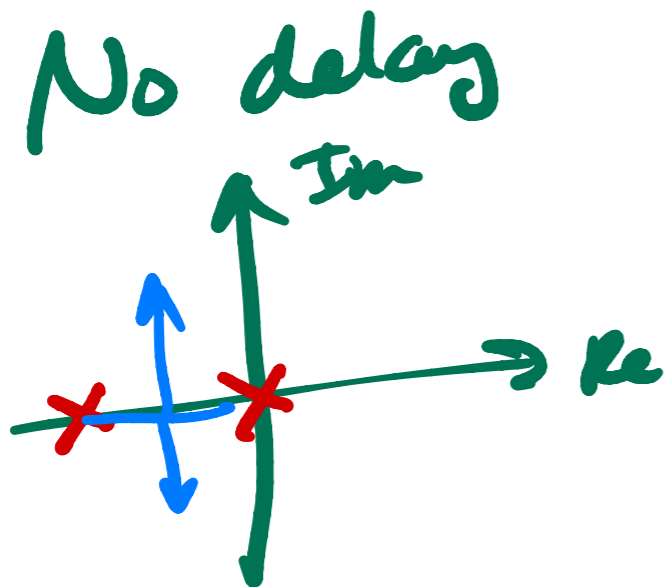
$$x = \frac{k e^{-sT}}{ms^2 + bs} (x_{wall} - e^{-sT} x)$$

$$x \left(1 + e^{-sT} \frac{k e^{-sT}}{ms^2 + bs} \right) = \frac{k e^{-sT}}{ms^2 + bs} x_{wall}$$

$$\frac{x}{x_{wall}} = \frac{k e^{-sT} \left(\frac{1}{ms^2 + bs} \right)}{1 + e^{-sT} \frac{k e^{-sT}}{ms^2 + bs}}$$

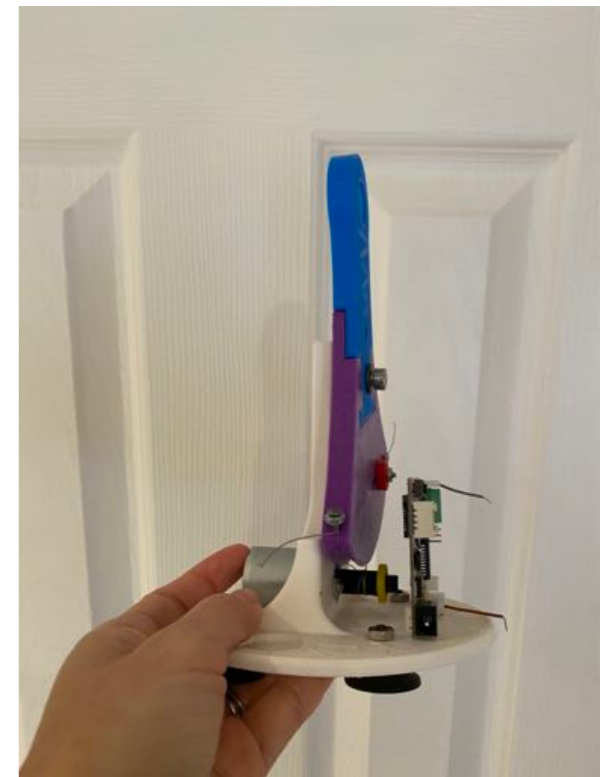
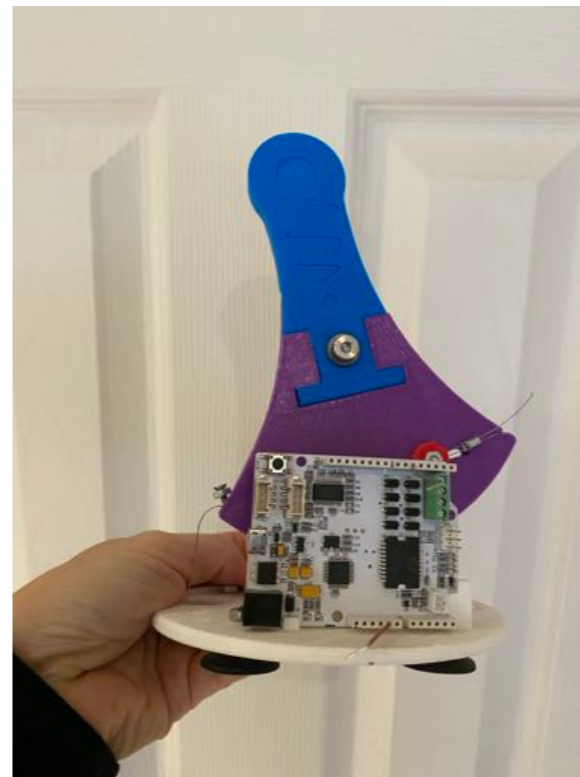
Char. eqn.

$$1 + \frac{K}{ms^2 + bs} \cdot \underbrace{\frac{1 - Ts}{1 + Ts}}_{(e^{-2sT})} = 0$$



Corrected from lecture!
 (I was using the
 higher-order approx.
 which gave complex
 two delays → this
 version uses the real
 poles & zeros from the
 true delay given
 above)

HAPKITS!!



Fill out address form **TODAY**
(see announcement on Canvas)

Reminders:

Assignment 4 due today

Look for Assignment 5 posted later today

Quiz will be discussed later — there are still a couple students who need to take it

Office Hours/Q&A with Allison until 10 am.

Question queue (see tab with today's date):

<https://tinyurl.com/HapticsAllison>